radial stress fields predicted from theoretical study. Fracture patterns similar to those shown in Fig. 4 have been observed from thermal loadings during experimental studies conducted at ADL.[6]

Applying this experimental evidence to an LNG storage tank excavated in rock, we can conclude that a minimum of perhaps six major cracks propagating to a distance of about one radius from the surface of the tank would be needed to relieve the stress field; if the crack propagation extends to only a $\frac{1}{2}$ radius, approximately twenty cracks could be expected.

This hypothesis assumes that the medium is essentially homogeneous and isotropic, and that the LNG does not penetrate into the larger cracks to any significant degree, which could result in further crack propagation. Actually, the fracture pattern will be altered, perhaps markedly, by LNG flow as well as by the specific characteristics of the rock strata, such as the existing crack and fault systems, schistosity, dip of the strata, anisotropy, etc.

Thermal Effects due to Cracking

Since it appears likely that large cracks will develop in frozen-rock, inground tank structures, the effect of such cracking on boiloff losses is of interest. These effects will tend to increase the predicted heat loads because the wetted surface area will be greater than anticipated.

Cracks occurring in or near the tank bottom, will be downward-directed and can produce a geysering[7] action that will introduce a very large heat leak into the tank.

If cracking occurs, we would expect boiloff to be at least 50% greater than that predicted for the uncracked case. The economics of peak shaving plants are very sensitive to boiloff losses. Therefore, cracking problems make rock inground storage less attractive than above-ground storage tanks.

Conclusion

The thermal performance of rock inground LNG storage tanks is adversely affected because major cracking occurs in the frozen rock and soil structure around the excavation. These cracks result in boiloff losses that we estimate to be at least 50% greater than would be predicted for an uncracked configuration. The cost savings possible with large-scale inground storage can be realized only if cracking is minimal or if an impermeable, inexpensive liner or sealer can be developed to prevent LNG flow into cracks. Techniques to maintain structural integrity during construction and operation must also be evaluated in the light of crack formation.

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Efficiency of a Pressurization Process

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ASTRONAUTS carry their air or oxygen supply with them in small pressure vessels that will have to be refilled periodically from larger containers of compressed gas. Filling of small pressure vessels from large supply containers also has more mundane applications (e.g., the charging of a shock-tube driver section).

The inefficiency of the procedure of charging vessels from larger supply containers is due to the fact that the gas in the supply vessel becomes unavailable when its pressure falls below the minimum required pressure of the vessel being charged. By using a second supply container to top off the vessel being charged, more of the gas in the first supply vessel becomes available. If several supply containers are used sequentially, it is clear that if the first supply container were nearly empty when the last fell below the required pressure, the charging procedure would make efficient use of the gas in each of the supply containers.

Although this type of charging procedure has been used for many years, it appears that no analysis of it has been published. This Note presents an analysis for an array of an arbitrary number of supply vessels, with equal volume and equal initial pressure, and an arbitrary number of chargings of smaller vessels, all with the same volume and initial pressure. The charging is done slowly, so conditions may be assumed to be isothermal. This analysis shows the effects of various parameters on the efficiency of gas utilization. Certain other useful results also are obtained.

Consider the pressurization of a number of vessels all with volume V_0 initially at pressure P_0 sequentially from an arbitrary number of supply vessels all with volume V^0 initially at a pressure P^0. For an isothermal expansion of a perfect gas the pressure of a supply vessel (and the pressure in the receiver vessel which it is charging since they are equal) is related to the supply vessel pressure and the receiver vessel pressure before these two pressures were equalized. Equating the mass lost by the supply container to that gained by the receiver vessel and employing the equation of state for a perfect gas the expression for the pressure of the supply and receiver vessels is obtained as

$$V^0(P_m{}^{n-1} - P_m{}^n) = V_0(P_m{}^n - P_{m-1}{}^n) \quad (1)$$

where

$P_m{}^{n-1}$ = pressure of supply vessel m after it has been used $n - 1$ times (initial pressure of m for the nth use)

$P_{m-1}{}^n$ = pressure of supply vessel $m - 1$ after it has been used n times (initial pressure of the vessel being charged by m on its nth use)

$P_m{}^n$ = the pressure of supply vessel m after its nth use and the pressure of the nth vessel that has been charged by m.

The preceding difference equation may be put into the standard form

$$P_m{}^n - \alpha P_m{}^{n-1} = (1 - \alpha)P_{m-1}{}^n \quad (2)$$

where

$$\alpha = V^0/(V^0 + V_0) = \text{volume ratio} < 1$$

For $m = 1$, the case of a single supply vessel, and since $P_0{}^n$ is

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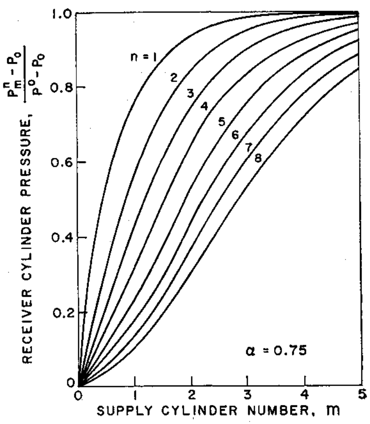

Fig. 1 Receiver cylinder pressure.

taken to be the same for each receiver cylinder namely, P_0,

$$P_1^n - \alpha P_1^{n-1} = (1 - \alpha)P_0 \quad (3)$$

The solution to the difference equation for this case is

$$P_1^n = (P^0 - P_0)\alpha^n + P_0 \quad (4)$$

For $m = 2$, an array of two supply vessels,

$$P_2^n - \alpha P_2^{n-1} = (1 - \alpha)P_1^n \quad (5)$$

combining Eqs. (4) and (5)

$$P_2^n - \alpha P_2^{n-1} = (1 - \alpha)[(P^0 - P_0)\alpha^n + P_0] \quad (6)$$

the solution of Eq. (6) is

$$P_2^n = (P^0 - P_0)\alpha^n[1 + n(1 - \alpha)] + P_0 \quad (7)$$

In a similar fashion the solutions for $m = 3$, 4, etc. can be found and by mathematical induction the general solution is found as

$$\frac{P_m^n - P_0}{P^0 - P_0} = \alpha^n \sum_{i=1}^{m} \frac{(n + i - 2)!}{(n - 1)!(i - 1)!} (1 - \alpha)^{i-1} \quad (8)$$

This expression yields the pressure of any supply vessel (m) at any stage of its use (n) and the pressure of any of the receiver vessels (n) at any stage of the charging procedure (m).

Figure 1 shows $(P_m^n - P_0)/(P^0 - P_0)$ vs supply vessel number (m) with the receiver vessel number (n) as a parameter for $\alpha = 0.75$. For the particular volume ratio considered, this graph can be used to determine the pressure in any one of the receiver vessels (n) after having been charged from some number of supply vessels (m). Alternatively, this figure gives the pressure of any supply vessel (m) as a function of the number of times it has been used (n).

We can define the gas utilization efficiency η as the amount of gas removed from a supply vessel before it is taken out of the array compared to its initial content of gas

$$\begin{aligned} \eta &= (P^0 - P_1^N)/P^0 \\ &= (P^0 - P_0)/P^0[1 - (P_1^N - P_0)/(P^0 - P_0)] \end{aligned} \quad (9)$$

where P_1^N is the pressure of the first supply vessel in the array when it is removed. This vessel is removed when the pressure in the last vessel in the array has dropped below the minimum required pressure of a receiver vessel. This minimum required pressure will be denoted by P_M^N and the number of the last supply cylinder in the array by M. From Eq. (8)

$$\frac{P_M^N - P_0}{P^0 - P_0} = \alpha^N \sum_{i=1}^{M} \frac{(N + i - 2)!}{(N - 1)!(i - 1)!} (1 - \alpha)^{i-1} \quad (10)$$

and

$$(P_1^N - P_0)/(P^0 - P_0) = \alpha^N \quad (11)$$

where N is the number of receiver vessels that have been charged when it becomes necessary to remove the first supply vessel and add a fresh one to the end of the array. In theory it is possible to solve Eq. (10) for N and using that value of N calculate P_1^N from Eq. (11) and thus determine the efficiency η from Eq. (9). However, this cannot be done, since Eq. (10) is transcendental in N.

It is possible to determine the gas utilization efficiency by what amounts to a graphical solution of Eqs. (10) and (11). Returning to Fig. 1 it can be seen that the pressure in the first supply vessel ($m = 1$) can be read from the figure for the same number of uses at which the pressure in last supply vessel reaches P_M^N. This technique has been applied to Fig. 1 and the results are shown in Fig. 2 where the efficiency $\eta[P^0/(P^0 - P_0)]$ is plotted as a function of the number of supply vessels in the array M for a number of values of the minimum required receiver pressure $(P_M^N - P_0)/(P^0 - P_0)$.

The following conclusions can be drawn from the results shown in Fig. 2. 1) Gas utilization efficiency increases rapidly with increased number of supply vessels. 2) A lower minimum required final pressure for the receiver vessels leads to a higher efficiency. 3) The number of supply cylinders required to obtain a given efficiency increases with increasing minimum required final pressure.

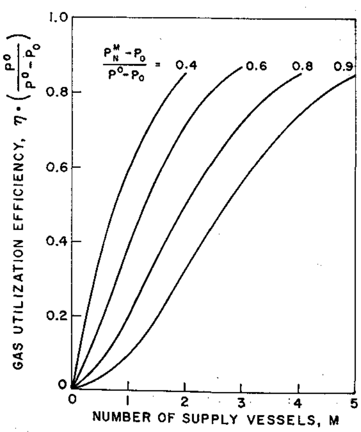

Fig. 2 Gas utilization efficiency.

Bondline Temperature of a Two-Layer Slab Subjected to Aeroheating—Graphical Solution

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Nomenclature

c_p = specific heat at constant pressure, Btu/lb-°F
F_0 = dimensionless time (Fourier modulus), $\alpha\theta/l^2$; F_{0f} = value $(\alpha\theta_f/l^2)$ at which the bondline temperature is to be determined
$f(x,\theta)$ = $T(x,\theta)$ (temperature as function of depth and time), °F
f' = $\partial T/\partial F_0$, °F
$f'(l,F_0 - \lambda)$ = "kernel" in Duhamel integral in Eq. (4)

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